On the Utility of Frequency Reuse in Cognitive Radio Channels

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***Abstract*— We consider the generalized cognitive radio channel where the secondary user is allowed to reuse the frequency during the *active* periods of the primary user, as long as the primary rate remains the same. In this setting, the optimal power allocation policy with a single antenna secondary transmitter (and receiver) is explored. Interestingly, we show that the offered gain resulting from the frequency reuse during the active periods of the spectrum disappears in both the low and high signal-to-noise ratio (SNR) regimes. This drawback, however, is shown to disappear with multi-antenna nodes by using simple zero-forcing strategies at both ends of the secondary channel.**

I. Background

In the classical cognitive radio set-up, the secondary users must first sense the wireless channel to determine the unused parts of the spectrum. Those users will then transmit their own messages during these white spaces in order to increase the overall spectral efficiency. In other words, the cognitive radios can only transmit over those particular frequency bands (or time intervals) which the licensed (primary) users are not transmitting.

In contrast to the classical cognitive radio approach, recent studies have introduced cognitive channels in which the secondary user exploits the active areas in the spectrum (i.e., simultaneously transmits with the primary users) as long as certain constraints are satisfied [1]–[4]. In the extreme case, where the primary user is willing to accommodate the needs of the secondary user, one can easily envision cooperation strategies where the *two* users can benefit [1]. Interestingly, even in the other extreme, where the primary user is ignorant of the secondary user presence, it was argued recently that frequency reuse is possible at secondary nodes during active primary period [3]. Here, we focus on the latter approach and revisit the conclusion drawn in [3] under a more realistic assumption on the secondary user side information in a generalized setting that allows the frequency re-use at the secondary users during both active and silent periods of the spectrum.

In particular, we consider a four-terminal network, in which the primary transmitter and receiver are Node 1 and Node 3, respectively; whereas their secondary counterparts are Node 2 and Node 4. All nodes are assumed to be half-duplex and the transmitters are limited by individual long-term average power constraints. In the classical cognitive radio channel, the secondary user is only allowed to transmit during **state 1**, in which the primary transmitter is silent. Here, we consider the *generalized* cognitive radio, where we allow the secondary user to transmit also in **state 2**, in which the primary user is active, as long as the following *coexistence constraints* are satisfied [4].

- Primary link has the same structure (encoder and decoder) as in the non-cognitive network.
- Primary users have the same performance (instantaneous achievable rate) as in the non-cognitive network.

The fundamental difference between our work and [4] is the relaxation of the unrealistic assumption that the secondary transmitter knows *a-priori* the signal to be transmitted over the primary link in the generalized setting above. In fact, by explicitly accounting for the time needed to decode the primary messages, at the single-antenna secondary transmitter, it is shown in the sequel that the gain offered by frequency re-use in state 2 disappears in the high and low SNR regimes. Furthermore, by equipping the secondary transmitter and receiver with multiple antennas, we show how this problem can be overcome in the high SNR regime.

II. SISO Cognitive Channel

We adopt the asymptotic assumption of $M \to \infty$ blocks with $N \to \infty$ channel uses per block. It is further assumed that the primary transmitter is silent, i.e., in state 1, in any particular block with probability p and the cognitive user is informed *a-priori* with **only** the states of the different blocks. Mathematically, we denote the instantaneous cognitive rate during state 1 and state 2 as $R_1(P_1)$ and $R_2(P_2)$, respectively. We also assume that the power of the secondary user linearly scales with the power of the primary user and denote P and βP as the total (long-term) average power constraints of the cognitive and primary transmitters, respectively. Thus, in this setting, the following cognitive rate is achievable if the coexistence constraints are satisfied with a choice of power allocation parameter $t \in [0,1]$:

$$R = \max_t \left\{ pR_1\left(\frac{P(1-t)}{p}\right) + (1-p)R_2\left(\frac{Pt}{(1-p)}\right)\right\} \tag{1}$$

In this section, we analyze this power allocation problem under the assumption of a single-input-single-output (SISO) cognitive link. The main hurdle now is to identify the optimal coding strategy when the secondary transmitter is re-using

the active primary period. Instead of pursuing this problem, which appears intractable at the moment, we assume that the system is in the low-interference-gain regime and the cognitive transmitter will implement the scheme proposed in [4] during state 2: It will first decode the primary message[1] in $\lceil \alpha N \rceil$ channel uses, and then, the cognitive transmitter will send its own message using dirty paper coding [5]. In order to maintain a fixed instantaneous rate of primary link, only a fraction of the available power will be allocated to this signal. The cognitive transmitter will use the remaining power to cooperate with the primary user in forwarding its message. The power allocation should be judiciously chosen such that the cooperation benefit will *exactly* compensate for the interference caused by the secondary signal. Under these assumptions, the optimal power control policy, for the secondary user, is given by

Theorem 1: The achievable rate of the SISO cognitive link using the Decode-Forward-Dirty Paper Code Scheme can be denoted as follows:

$$\begin{aligned} R &= \max_{t} \left\{ p \log\left(1+\frac{|c_{24}|^2 P(1-t)}{p}\right) \right. \\ &\quad \left. +(1-\alpha)(1-p)\log\left(1+\frac{|c_{24}|^2 uPt}{(1-\alpha)(1-p)}\right)\right\}, \end{aligned} \tag{2}$$

where

$$\begin{aligned} \alpha &= \frac{\log\left(1+\frac{|c_{13}|^2\beta P}{(1-p)}\right)}{\log\left(1+\frac{|c_{12}|^2\beta P}{(1-p)}\right)} \\ u &= 1-\left(\frac{|c_{13}|\sqrt{\beta}\left(-\sqrt{(1-\alpha)}(1-p)\mp\sqrt{\delta}\right)}{|c_{23}|\sqrt{t}\left(1-p+|c_{13}|^2\beta P\right)}\right)^2 \\ \delta &= (1-\alpha)(1-p)^2+P|c_{23}|^2 t\left(1-p+|c_{13}|^2\beta P\right), \end{aligned} \tag{3}$$

and $u \in [0,1]$.

Moreover, in the limits $P \to 0$ or $P \to \infty$, the scheme reduces to the classical cognitive channel where the secondary transmitter is only active in the silent periods of the primary link (i.e., the optimal point for (2) is $t = 0$).

Proof: Please refer to Appendix A. ∎

The previous claims are validated numerically in Fig. 2, which uses the linearized channel model of Fig. 1 (with a path loss exponent of 2). This figure shows the gain offered by the generalized cognitive radio (GCR), as compared with the classical cognitive radio (CCR), to be significant only in the medium SNR regime. The culprit behind our negative result, in the high SNR regime, is the decoding time required by the cognitive transmitter to figure out the primary message which dominates the whole block asymptotically (i.e., $\alpha \to 1$ as $P \to \infty$). While our result pertains only to the scheme proposed in [4], we remark that this approach is optimal among the class of SISO cognitive schemes that require

[1]It is assumed that the channel coefficients are known at the cognitive radios and the channel between the primary transmitter and the cognitive transmitter is stronger than the channel between primary users.

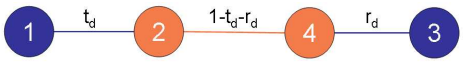

Fig. 1. Linear system model: Distance between transmitters and receivers are denoted as t_d and r_d, respectively.

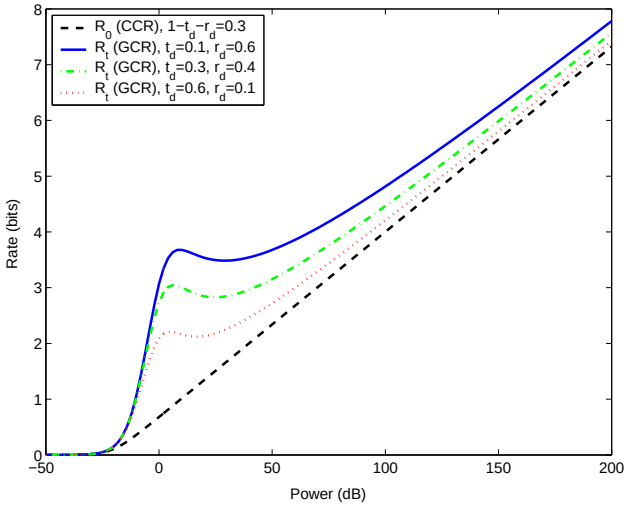

Fig. 2. Simulation results for the SISO cognitive link with the linear system model ($p = 0.1, \beta = 1$).

decoding of the primary message at the secondary transmitter in the low-interference-gain regime ($|c_{23}| \leq |c_{24}|$) [4], [6]. We further note that, for AWGN systems that implement the *coexistence constraints*, there is no known SISO cognitive schemes that allow the frequency reuse of the active primary channel without needing to decode the primary signal at cognitive radios.

III. MISO Cognitive Channel

In an attempt to overcome the negative result reported in the previous section, we equip the secondary transmitter with a second antenna (the coefficient between the k^{th} at node i and node j will be referred to as $c_{ij,k}$). Having multiple antennas, the cognitive transmitter can Zero-Force (ZF) its secondary message at the primary receiver, and hence it does not need to forward the primary message anymore. However, the cognitive transmitter still needed to decode the primary message in order to perform dirty paper coding. We call this scheme the Decode-ZF-Dirty Paper Code Scheme whose achievable rate, along with the optimal power allocation policy, is characterized in the following result.

Theorem 2: The achievable rate of the MISO cognitive link using the Decode-ZF-Dirty Paper Code Scheme is given by:

$$R = \max_{t} \left\{ p \log\left(1+\left(|c_{24,1}|^2+|c_{24,2}|^2\right)\frac{P(1-t)}{p}\right) + (1-\alpha)(1-p)\log\left(1+|a_1 c_{24,1}+a_2 c_{24,2}|^2 \frac{Pt}{(1-\alpha)(1-p)}\right)\right\},$$

where

$$\alpha = \frac{\log\left(1+|c_{13}|^2\frac{P\beta}{(1-p)}\right)}{\log\left(1+\left(|c_{21,1}|^2+|c_{21,2}|^2\right)\frac{P\beta}{(1-p)}\right)} \tag{4}$$

And, similar to the SISO case, the generalized cognitive radio will reduce to the classical one in the asymptotic scenarios $P \to 0$ and $P \to \infty$.

Proof: Please refer to Appendix B. ■

IV. MIMO Cognitive Channel

The next step is to equip the cognitive receiver with an additional antenna (the coefficient between the k^{th} at node i and m^{th} at node j will be referred to as $c_{ij,km}$). The additional antenna allows the cognitive receiver to ZF the primary signal without needing to employ dirty paper coding. This way, one can avoid the need to decode the primary signal at the cognitive transmitter in state 2. In summary, the proposed scheme for MIMO cognitive channel decomposes into two ZF stages; namely 1) The cognitive transmitter ZF its own signal at the primary receiver and 2) The cognitive receiver ZF the primary signal. The power levels used by the cognitive transmitter are then obtained from the water-filling solution. More precisely, the following result characterizes the achievable rate and optimal power allocation policy for this scheme.

Theorem 3: The achievable rate of the MIMO cognitive link using the ZF Scheme is given by:

$$R = \max_{t} \left\{ pR_1 + (1-p)R_2 \right\}, \tag{5}$$

where

$R_1 = \log\left(1 + \gamma\lambda_1 \frac{P(1-t)}{p}\right) + \log\left(1 + (1-\gamma)\lambda_2 \frac{P(1-t)}{p}\right)$

$R_2 = \log\left(1 + \frac{|c_{eff}|^2}{|c_{41,1}|^2 + |c_{41,2}|^2} \frac{Pt}{(1-p)}\right)$,

and γ, λ_1, λ_2 are the parameters of the solution of waterfilling problem for MIMO cognitive link; and $c_{eff} = -c_{24,11}c_{41,2}a_1 - c_{24,21}c_{41,2}a_2 + c_{24,12}c_{41,1}a_1 + c_{24,22}c_{41,1}a_2$.

Moreover, this scheme outperforms the classical cognitive radio approach in the high power region if $c_{eff} \neq 0$ and achieves the optimal multiplexing gain of the corresponding interference channel in the high power region.

Proof: Please refer to Appendix C. ■

Here, we note that the cognitive MIMO gain obtained by exploiting the active spectrum will disappear in the high SNR regime if c_{eff} is zero. This situation corresponds to *singular* channel in which the transmitter (or receiver) ZF cancels the secondary signal, as seen by the secondary receiver. Finally, Fig. 3 reports the performance gain of the proposed generalized MIMO cognitive radio, as compared with the classical approach. To generate this figure, we used independent zero-mean circularly symmetric complex Gaussian channel coefficients, each having unit variance. The gain offered by the proposed approach is evident in the figure. Moreover, this gain in degrees of freedom, i.e., slope of the curve, is shown to approach $1-p$ (the probability of the active state) as the SNR grows.

V. Conclusion

In this work, we investigated the gain that can be leveraged from re-using the active areas in the frequency (time) spectrum of cognitive channels. It was argued that this gain is limited, in the high and low SNR regimes, when the cognitive nodes are equipped with only single antennas. The limiting factor, in this scenario, was the need to decode the primary signal at the cognitive transmitter in order to satisfy the *coexistence constraints*. With the employment of multiple antennas at the cognitive nodes, we have shown how to overcome this limitation in the high SNR regime by using transmitter and receiver ZF. The proposed ZF strategy was also shown to achieve the optimal multiplexing gain of the corresponding interference channel.

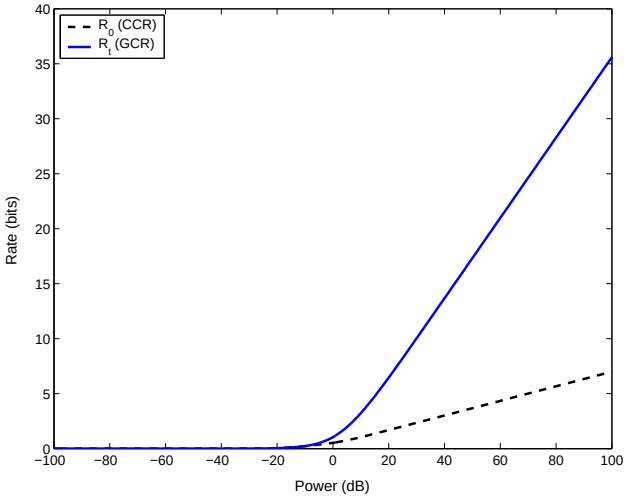

Fig. 3. Simulation results for the MIMO cognitive link with Rayleigh fading channel coefficients ($p = 0.1, \beta = 1$).

Appendix

A. Proof of Theorem 1

During state 1, cognitive transmitter will transmit the cognitive message with an instantaneous rate of R_1.

$$R_1 = \log\left(1 + \frac{|c_{24}|^2 P(1-t)}{p}\right) \tag{6}$$

Then, after the first α fraction of state 2, if $|c_{12}| > |c_{13}|$, the secondary transmitter can decode the primary signal, which is a circularly symmetric complex Gaussian random variable $\sim \mathcal{CN}\left(0, \frac{\beta P}{(1-p)}\right)$, where

$$\alpha = \frac{\log\left(1 + \frac{|c_{13}|^2 \beta P}{(1-p)}\right)}{\log\left(1 + \frac{|c_{12}|^2 \beta P}{(1-p)}\right)} \tag{7}$$

During the remaining fraction of state 2, secondary transmitter will form the signal $X_2[n] = \sqrt{ut} D_2[n, X_1[n]] + \frac{c_{23}^*}{|c_{23}|} e^{j\angle c_{13}} \sqrt{\frac{(1-u)t}{\beta(1-\alpha)}} X_1[n]$, where $u \in [0,1]$. Here, the first part of the message is the dirty paper coded cognitive signal with known interference, i.e., the primary signal with a scale factor, and the second part is the primary signal, which is scaled according to power constraints and phase shifted to add-up coherently with the primary signal at the primary receiver. At this point, we remark that the resulting dirty-paper code is

independent of the primary signal and hence can be considered as noise at the primary receiver. See [4], [5] for details.

As the secondary transmitter uses the above signaling scheme, receivers in the system (i.e., Node 3 and Node 4) will receive the following signals:

$$\begin{aligned} Y_3[n] &= c_{13}X_1[n] + c_{23}X_2[n] + Z_3[n] \\ Y_4[n] &= c_{14}X_1[n] + c_{24}X_2[n] + Z_4[n] \end{aligned}$$

where $Z_i \sim C\mathcal{N}(0,1)$ at Node i, for $i = 3, 4$. Accordingly, the instantaneous rates of the primary link (R_p) and the cognitive link (R_c) can be represented as:

$$\begin{aligned} R_p &= \log\left(1 + \frac{\left(|c_{13}|\sqrt{\beta} + |c_{23}|\sqrt{\frac{(1-u)t}{(1-\alpha)}}\right)^2}{\frac{|c_{23}|^2 ut}{(1-\alpha)} + \frac{(1-p)}{P}}\right) \\ R_c &= \log\left(1 + \frac{|c_{24}|^2 uPt}{(1-\alpha)(1-p)}\right) \end{aligned} \quad (8)$$

where u is chosen such that $R_p = \log\left(1 + \frac{|c_{13}|^2\beta P}{(1-p)}\right)$ in order to satisfy the *coexistence constraints*. It follows that

$$\begin{aligned} u &= 1 - \left(\frac{|c_{13}|\sqrt{\beta}\left(-\sqrt{(1-\alpha)}(1-p) \mp \sqrt{\delta}\right)}{|c_{23}|\sqrt{t}\left(1 - p + |c_{13}|^2\beta P\right)}\right)^2 \quad (9) \\ \delta &= (1-\alpha)(1-p)^2 + P|c_{23}|^2 t\left(1 - p + |c_{13}|^2\beta P\right) \end{aligned}$$

, and $u \in [0,1]$. Finally, observing $R_2 = (1-\alpha)R_c$ and using R_1 and R_2 in (1) gives the achievable rate of the SISO cognitive link.

Now, for low SNR analysis, let's define the slope (S) of the rate (R) with respect to power (P) as in $R \approx \log(e)SP$, as $P \to 0$. Then, $S \approx \max_t \left\{|c_{24}|^2(1 - t + ut)\right\}$ and $S_0 \approx |c_{24}|^2$ as $P \to 0$, where S and S_0 are the SNR gains of generalized and classical cognitive radio, respectively. Since $0 \le t \le 1$ and $0 \le u \le 1$, the maximum of S/S_0 in the low power regime can occur at either "$t = 0$" or "$t \neq 0$ and $u = 1$". At this point, the latter case can not happen since u has to satisfy (3) with $P \neq 0$; and the former case will result in $R = R_0$. Therefore, at low SNR, cognitive transmitter will only use state 1 to transmit its message instead of exploring the spectrum opportunities in state 2.

For the high power region, one can denote the achievable rate of the cognitive link as

$$R = \max_t \left\{p\log\left(1 + \frac{|c_{24}|^2 P(1-t)}{p}\right) + (1-p)\Delta\right\},$$

where as P$\to \infty$, using (3), we have

$$\Delta \approx \frac{\log\left(\frac{|c_{12}|^2}{|c_{13}|^2}\right)}{\log(P)} \log\left(1 + \frac{2|c_{24}|^2\sqrt{t}\sqrt{\log(P)}}{\sqrt{\log\left(\frac{|c_{12}|^2}{|c_{13}|^2}\right)}|c_{13}||c_{23}|\sqrt{\beta}}\right)$$

Hence, $\Delta \to 0$ in the above equation for every choice of $t \in [0,1]$ as power increases. Therefore, the maximization in (2) has the solution $t = 0$ in the high power regime. This observation leads us to conclude that the generalized cognitive radio will behave as the classical one, which uses only silent periods of primary link, in the high power regime.

B. Proof of Theorem 2

During the silent period of the primary link, cognitive link is a simple MISO channel. Implementing transmit beamforming, the instantaneous rate of R_1 below is achievable.

$$R_1 = \log\left(1 + \left(|c_{24,1}|^2 + |c_{24,2}|^2\right)\frac{P(1-t)}{p}\right) \quad (10)$$

During the active primary period, primary message is also received at the cognitive transmitter with an instantaneous rate of $\log\left(1 + \left(|c_{21,1}|^2 + |c_{21,2}|^2\right)\frac{P\beta}{(1-p)}\right)$ using receive beamforming. Since the instantaneous rate of the primary link is $\log\left(1 + |c_{13}|^2\frac{P\beta}{(1-p)}\right)$, listening time fraction, i.e., α, will be

$$\alpha = \frac{\log\left(1 + |c_{13}|^2\frac{P\beta}{(1-p)}\right)}{\log\left(1 + \left(|c_{21,1}|^2 + |c_{21,2}|^2\right)\frac{P\beta}{(1-p)}\right)} \quad (11)$$

After decoding the primary message, cognitive transmitter can use the channel to transmit its own message while zero forcing its signal at the primary receiver. To accomplish ZF, cognitive transmitter will send a_1X_2 from its first antenna and a_2X_2 from its second antenna satisfying

$$\begin{aligned} |a_1|^2 + |a_2|^2 &= 1 \\ a_1c_{23,1} + a_2c_{23,2} &= 0 \end{aligned} \quad (12)$$

Hence, the instantaneous rate of the secondary link can be represented as $\log\left(1 + |a_1c_{24,1} + a_2c_{24,2}|^2\frac{Pt}{(1-\alpha)(1-p)}\right)$. Using these results in (1) gives the achievable rate expression for this scheme.

For the low power region analysis, the following approximations can be made:

$$\begin{aligned} R &\approx log(e)P\max_t\left\{(1-t)\left(|c_{24,1}|^2 + |c_{24,2}|^2\right)\right. \\ &\quad \left. + t|a_1c_{24,1} + a_2c_{24,2}|^2\right\}, \text{ as } P \to 0 \\ R_0 &\approx log(e)P\left(|c_{24,1}|^2 + |c_{24,2}|^2\right), \text{ as } P \to 0 \end{aligned} \quad (13)$$

, where R_0 is the rate of the classical cognitive radio. Here, the generalized cognitive approach is beneficial in the low power regime if $|c_{24,1}|^2 + |c_{24,2}|^2 < |a_1c_{24,1} + a_2c_{24,2}|^2$. However, this condition can not be satisfied. This can be shown by a simple contradiction. If the condition above were true for some coefficients a_1 and a_2, then the secondary transmitter can use this transmitting scheme also for the silent period of the primary link with a corresponding rate higher than capacity of 2x1 MISO case.

Secondly, in the high power region, one can represent the additive rate gain resulting from the utilization of the active period of the primary link as $\Delta = (1-\alpha)(1-p)\log\left(1 + |a_1c_{24,1} + a_2c_{24,2}|^2\frac{Pt}{(1-\alpha)(1-p)}\right)$. From this equation, one can readily conclude that $\frac{\Delta}{\log(P)} \to 0$ as $P \to \infty$, by inserting α from above and taking the limit. This observation gives the high SNR conclusion in the theorem and completes the proof.

C. Proof of Theorem 3

During state 1, since the primary transmitter is in silent mode, the cognitive link can fully utilize the channel as a 2x2 MIMO channel [7], where the channel coefficients can be represented as in the channel matrix below.

$$H = \begin{bmatrix} c_{24,11} & c_{24,21} \\ c_{24,12} & c_{24,22} \end{bmatrix} \quad (14)$$

Let's denote $E\left[X_2 X_2^H\right] = Q$ and $UQU^H = \tilde{Q}$, where we have the decomposition $H^H H = U^H \Lambda U$. Expressing the eigenvalues of $H^H H$ as λ_i's, we can denote the instantaneous rate during this state as

$R_1 = \log\left(1 + \gamma\lambda_1 \frac{P(1-t)}{p}\right) + \log\left(1 + (1-\gamma)\lambda_2 \frac{P(1-t)}{p}\right)$, for some $\gamma \in [0,1]$ satisfying

$$\tilde{Q} = \begin{bmatrix} \frac{\gamma P(1-t)}{p} & 0 \\ 0 & \frac{(1-\gamma)P(1-t)}{p} \end{bmatrix} \quad (15)$$

where $\tilde{Q}_{ii} = (\mu - \lambda_i^{-1})^+$ for $i = 1, 2$ and for some μ.

In the active period of the primary link, secondary transmitter will be in transmitting mode and it will be transmitting $a_1 X_2$ and $a_2 X_2$ from its first and second antenna, respectively, satisfying (12). During this state, cognitive receiver will receive the following signals at its antennas.

$Y_{4,1}[n] = c_{41,1}X_1[n] + (c_{24,11}a_1 + c_{24,21}a_2)X_2[n] + Z_1[n]$

$Y_{4,2}[n] = c_{41,2}X_1[n] + (c_{24,12}a_1 + c_{24,22}a_2)X_2[n] + Z_2[n]$

where $Z_i \sim C\mathcal{N}(0,1)$ at the i^{th} antenna of the secondary receiver, for $i = 1, 2$.

At this point, secondary receiver can form the signal $Y[n] = -c_{41,2}Y_{4,1}[n] + c_{41,1}Y_{4,2}[n] = c_{eff}X_2[n] + c_{41,1}Z_2[n] - c_{41,2}Z_1[n]$ in order to ZF the effect of primary transmitter, where $c_{eff} = -c_{24,11}c_{41,2}a_1 - c_{24,21}c_{41,2}a_2 + c_{24,12}c_{41,1}a_1 + c_{24,22}c_{41,1}a_2$. Cognitive receiver can decode the message of the cognitive transmitter from this processed output with an achievable instantaneous rate of

$$R_2 = \log\left(1 + \frac{|c_{eff}|^2}{|c_{41,1}|^2 + |c_{41,2}|^2} \frac{Pt}{(1-p)}\right) \quad (16)$$

Hence, using R_1 and R_2 above, the power allocation problem for MIMO cognitive link can be denoted as follows:

$$R = \max_t \left\{ pR_1 + (1-p)R_2 \right\} \quad (17)$$

Now, one can denote the classical cognitive radio case as R_0, where we choose $t = 0$ above, and observe the following approximations in the low SNR regime. Letting $P \to 0$ gives

$$R_0 \approx \log(e)P\left[\gamma\lambda_1 + (1-\gamma)\lambda_2\right] \quad (18)$$

$$R \approx \log(e)P \max_t \left\{\gamma\lambda_1 + (1-\gamma)\lambda_2 + t\psi\right\} \quad (19)$$

where $\psi = \frac{|c_{eff}|^2}{|c_{41,1}|^2 + |c_{41,2}|^2} - \gamma\lambda_1 - (1-\gamma)\lambda_2$. Therefore, there is a gain in the low SNR regime, if $\psi > 0$. However, if this condition were true for some coefficients a_1 and a_2, then the secondary link can use this transmitting scheme also for the silent period of the primary link with a corresponding rate greater than the capacity of 2x2 MIMO case. The fact that there is no gain in the low power regime follows from this contradiction.

Secondly, we will have the following approximations in the high power regime: As $P \to \infty$,

If $\gamma \neq 0$ and $\gamma \neq 1$, then $R_0 \approx 2p\log(P)$ and $R \approx (1-p)\log(P) + 2p\log(P)$

If $\gamma = 0$ or $\gamma = 1$, then $R_0 \approx p\log(P)$ and $R \approx (1-p)\log(P) + p\log(P)$

where we assumed $c_{eff} \neq 0$. Now, if we define the gain (G) in the high power regime as in $R \approx G\log(P)$, as $P \to \infty$, this result can be expressed as $G = G_0 + (1-p)$, where G_0 is the gain of the classical case.

For optimality, we first compute the degrees of freedom (DoF) of the whole system using the ZF Scheme. The proposed scheme is optimal during state 1 with a DoF of $2p$, if the channel matrices between users are not rank deficient. During the active period of the primary link, the achievable sum rate of the system scales as $2(1-p)\log(P)$ with the ZF Scheme above. Summing up, the achievable sum rate of the system scales as $2\log(P)$. Now, let's denote the number of antennas of Node i as A_i, for $i = 1, 2, 3, 4$. At this point, one can observe that capacity region of the cognitive MIMO channel that does not allow cooperation is contained in the capacity region of the MIMO interference channel, since the latter case does not have the *coexistence constraints* and it can always mimic the former one. DoF for the MIMO interference channel has recently shown in [8], which gives the DoF of the setting in this sequel as $\min\{A_1 + A_2, A_3 + A_4, \max(A_1, A_4), \max(A_2, A_3)\} = 2$. Hence, by observing that the achievable sum rate of the system scales with a maximum multiplexing gain, one can conclude that the ZF Scheme is optimal among non-cooperative schemes in terms of the gain in the high power region. However, we remark here that the multiplexing gain of the MIMO interference channel with transmit cooperation is still an open problem. Thus, it may be possible to achieve larger multiplexing gains with unidirectional cooperative schemes.

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