On the degrees of freedom of the cognitive broadcast channel

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***Abstract*—Cognitive broadcast channel, where two multi-antenna transmitters communicate with their respective receivers, is considered. One of the transmitters is said to be cognitive (secondary) as it is assumed to know the messages of the other (primary) transmitter non-causally. The goal is to design cooperative schemes between the two transmitters, which impose only minimal changes to the primary broadcast channel (compared to the non-cognitive scenario). Towards this end, an achievable scheme is provided under which both intra cell and inter cell interferences at the primary receivers are aligned. The interference at the secondary receivers, on the other hand, is canceled by dirty paper coding. The corresponding achievable region and an outer bound region are provided in terms of the degrees of freedom (DoF) metric. Special cases shows the optimality of the proposed scheme in the high SNR regime for those cases. We also illustrate the advantage of the cognitive cooperation over the non-cognitive system by proving that the achieved sum DoF is strictly larger than the non-cognitive case.**

I. Introduction

Cognitive cooperation has been proposed to improve the performance of the existing (single frequency, single protocol) wireless systems [1], [2]. In this work, we consider the application of cognitive cooperation to a cellular network with two base stations where each base station (BS) serves a number of mobile users in its cell. Under our considered setting, the messages of one cell's (henceforth the non-cognitive or primary cell) is assumed to be *non-causally* known at the other cell's (henceforth the cognitive or secondary cell) BS. The validity of this assumption hinges on the fact that in many cellular applications the base stations can be assumed to be connected by high capacity links. While the cognitive BS is assumed to be able to adapt its signaling strategy, we assume that few changes can be made to the communication scheme of the primary cell.

For either the primary or secondary users, the signals transmitted for the opposite cell is interference, which is also referred to as *inter cell interference*. In general, inter cell interference can significantly degrade the throughput of cellular systems. A similar problem arises in a K-user MIMO interference channel where the data of indirect users interfere with the direct ones. An important technique proposed to mitigate these effects is interference alignment (IA) [3], [4]. Heuristically speaking, IA *aligns* all the unwanted (interfering) signals to certain dimensions and makes it possible for the intended messages to be communicated over the interference free ones. The IA technique proposed in [3] assumes the availability of perfect and global channel state information (CSI) at all the transmitters. However, the overhead introduced by the training sequences and feedback to obtain global CSI can significantly deteriorate the system throughput.

In a previous work ([5]), we considered the same setting with the assumption that the available CSI at the primary BS consists only of its own cell. In this paper, we consider a new scenario of CSI availability. Let us refer to the set of channels from the primary BS to the receivers in the primary cell as the primary *intra cell channels* and the set of channels from the primary BS to the receivers in the secondary cell as the primary *inter cell channels*. In this work, we assume that the primary BS has full knowledge of both primary inter cell and intra cell CSIs. As in [5], we assume the cognitive BS to be equipped with full CSI information.

In the following, we propose a novel interference management scheme for this new setting: Each mobile user experiences two types of interference: 1) The signals intended for the mobile users in the opposite cell or the *inter cell* interference, 2) The signals intended for the other mobile users in its own cell which cause *intra cell* interference. In the proposed scheme, the intra cell interference for each primary mobile user is *aligned* to the linear space spanned by the inter cell interference using the IA technique proposed in [6]. By doing so, they can be canceled at the same time. Utilizing the full CSI availability at the cognitive BS, the inter cell interference from the primary BS to cognitive users is canceled using dirty paper coding (DPC) [7].

An outer bound on the sum DoF region is also derived by employing the techniques of [8] and [9]. Using the derived outer bound, it is established that the proposed scheme is optimal under some special cases.

The rest of the paper is organized as follows. Section II introduces the system model and problem formulation. In Section III, we explain our proposed signaling scheme in detail. The outer bounds and special cases are studied in Section IV, and, finally, the paper is concluded in Section V.

II. System model

A cellular system with one primary and one secondary base stations (denoted respectively by $\mathcal{P}$ and $\mathcal{S}$) is considered.

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The primary and secondary base stations serve K_P (denoted by $\mathcal{P}_1, \mathcal{P}_2, \cdots, \mathcal{P}_{K_P}$) and K_S (denoted by $\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_{K_S}$) mobile users in their cells respectively.

Let us denote the messages intended for primary users by $W_{\mathcal{P}_1}, W_{\mathcal{P}_2}, \cdots, W_{\mathcal{P}_{K_P}}$, and for the secondary users by $W_{\mathcal{S}_1}, W_{\mathcal{S}_2}, \cdots, W_{\mathcal{S}_{K_S}}$. The total power available at the base stations is denoted by ρ. Rates of $R_{\mathcal{P}_i}(\rho)$, $R_{\mathcal{S}_j}(\rho)$ are said to be achievable with power ρ, if there exists a coding scheme to reliably communicate messages of sizes $|W_{\mathcal{P}_i(\rho)}| = 2^{nR_{\mathcal{P}_i}(\rho)}$ and $|W_{\mathcal{S}_j}(\rho)| = 2^{nR_{\mathcal{S}_j}(\rho)}$ to mobile users $\mathcal{P}_i$ and $\mathcal{S}_j$, where n is the number of channel uses. Let us denote $\big(R_{\mathcal{S}_1}(\rho), \cdots, R_{\mathcal{S}_{K_S}}(\rho); R_{\mathcal{P}_1}(\rho), \cdots, R_{\mathcal{S}_{K_P}}(\rho)\big)$ by $\mathcal{R}(\rho)$ and the set of all achievable rate tuples at power ρ by $\mathcal{C}(\rho)$. Following the notation introduced in [8], the sum DoF in the secondary and primary cells d_S and d_P are respectively defined by:

$$d_S = \limsup_{\rho\to\infty} \left\{ \sup_{\mathcal{R}(\rho)\in\mathcal{C}(\rho)} \left\{ \sum_{j=1}^{K_S} R_{\mathcal{S}_j}(\rho) \right\} \frac{1}{\log(\rho)} \right\} \tag{1}$$

and

$$d_P = \limsup_{\rho\to\infty} \left\{ \sup_{\mathcal{R}(\rho)\in\mathcal{C}(\rho)} \left\{ \sum_{i=1}^{K_P} R_{\mathcal{P}_i}(\rho) \right\} \frac{1}{\log(\rho)} \right\} \tag{2}$$

The set of all achievable pairs of (d_S, d_P) is denoted by $\mathcal{D}_{\text{sum}}$, and is referred to as sum DoF region. We also refer to $d_S + d_P$ as the *total sum DoF*.

Throughout the paper, it is assumed that the secondary base station non-causally knows the messages intended for the primary users, i.e., at $\mathcal{S}$, the messages $\{W_{\mathcal{P}_i}(\rho)\}_{i=1}^{K_P}$ are known prior to transmission. The users are also assumed to be equipped with multiple antennas. Let m_P and m_S denote the number of antennas at the primary and secondary base stations, n_P and n_S be the number of antennas at the primary and secondary users. The explained model is referred to as a $\{m_P, m_S, n_P, n_S, K_P, K_S\}$ cognitive system.

In general, using time and/or frequency expansions (by multiple fading blocks and/or multiple OFDM subcarriers) we can generate L extra dimensions on each user. In that case, the number of available dimensions on each node is equal to: $M_P = L \times m_P$, $M_S = L \times m_S$, $N_P = L \times n_P$ and $N_S = L \times n_S$ respectively.

The received signal at user $\mathcal{P}_i$ is given by:

$$\mathbf{y}_{\mathcal{P}_i} = \mathbf{H}_{\mathcal{P}_i}\mathbf{x}_P + \mathbf{H}'_{\mathcal{P}_i}\mathbf{x}_S + \mathbf{z}_{\mathcal{P}_i}, \tag{3}$$

where $\mathbf{H}$'s represents the *extended* MIMO channel coefficients to the users and $\mathbf{H}'$'s are the extended MIMO channel from the cognitive base station. For both, $\mathbf{H}$ and $\mathbf{H}'$ the subscript denotes the receiver. For example, $\mathbf{H}_{P_i}$ which is a matrix of size $N_P \times M_P$ and $\mathbf{H}'_{P_i}$ which is $N_P \times M_S$ are the channels from the primary and secondary base stations to the i-th primary user respectively. $(\mathbf{x}_P)_{M_P\times 1}$ and $(\mathbf{x}_S)_{M_S\times 1}$ denote the signals transmitted from the primary and secondary base stations and $\mathbf{z}$'s are the zero mean unit variance i.i.d. additive white Gaussian noise.

Similarly, the signal received at the j-th secondary mobile user will be:

$$\mathbf{y}_{\mathcal{S}_j} = \mathbf{H}_{\mathcal{S}_j}\mathbf{x}_S + \mathbf{H}'_{\mathcal{S}_j}\mathbf{x}_P + \mathbf{z}_{\mathcal{S}_j} \tag{4}$$

In this paper, it is assumed that the channel coefficients are drawn independently from a continuous distribution and thus the channel matrices are full rank almost surely. Also, we assume the cognitive base station to have full CSI knowledge. The primary base station is assumed to know its own cell's CSI as well as the inter cell CSI. That is, all $\mathbf{H}_{\mathcal{P}_i}$'s and $\mathbf{H}'_{\mathcal{P}_i}$'s are known at the primary BS.

III. Main Result

In this section, we present an achievable region for the sum DoF in the primary and secondary cells under the explained system model.

Theorem 1: For a $\{m_P, m_S, n_P, n_S, K_P, K_S\}$ cognitive system, denote the set of all pairs (d_S, d_P) for which

$$\begin{cases} 0 \le d_S \le \min\big\{K_S n_S, m_S\big\} \\ 0 \le d_P \le \min\big\{K_P(n_P - d_S)^+, (m_P + m_S - d_S)\big\}, \end{cases}$$

by $\mathcal{D}_{\text{sum}}^{in}$. Then, $\mathcal{D}_{\text{sum}}^{in} \subseteq \mathcal{D}_{\text{sum}}$.

Proof: First let us assume d_S to be a rational number and denote its irreducible form by

$$d_S = \frac{S}{L}.$$

$L \in \mathbb{Z}^+$ extra dimensions in time or frequency (through multiple fading blocks or OFDM subcarriers) are generated. The achievable sum DoF in the secondary cell cannot exceed m_S, and we always have: $S \le M_S$.

Because the cognitive BS has full CSI knowledge including the CSI from itself to the primary users and also knows all the primary messages it can *lend* $(M_S - S)^+$ of its available dimensions which are not used for data transmission of the secondary users to the primary BS. More specifically, for each primary user $\mathcal{P}_i$ we "append" the channels from the first $(M_S - S)$ dimensions of the cognitive BS (corresponding to the first $(M_S - S)$ columns of $\mathbf{H}'_{\mathcal{P}_i}$) to $\mathbf{H}_{\mathcal{P}_i}$ to form a new matrix $\bar{\mathbf{H}}_{\mathcal{P}_i}$ of size $N_P \times (M_P + M_S - S)$. Those columns are deleted from $\mathbf{H}'_{\mathcal{P}_i}$ and the $N_P \times S$ matrix $\bar{\mathbf{H}}'_{\mathcal{P}_i}$ is formed.

Similarly, by allocating first $(M_S - S)$ dimensions of the secondary BS to data transmission of the primary users, the data transmitted from those can be thought of as inter cell interference on each secondary user. In other words, we can form the matrices $\bar{\mathbf{H}}_{\mathcal{S}_j}$ and $\bar{\mathbf{H}}'_{\mathcal{S}_j}$ of sizes $N_S \times S$ and $N_S \times (M_P + M_S - S)$ capturing the matrices which carry intra cell and inter cell data to user $\mathcal{S}_j$.

Let us specify the transmission scheme to the primary users first. For each $(\bar{\mathbf{H}}'_{\mathcal{P}_i})_{[N_P\times S]}$, the primary base station calculates r linearly independent normalized basis vectors of its null space denoted by $\mathbf{u}^1_{\mathcal{P}_i}, ..., \mathbf{u}^r_{\mathcal{P}_i}$ where $r = (N_P - S)^+$. The goal of $\mathcal{P}$, is to find zero forcing beam forming vectors such that on each primary user the intra cell interference is *aligned* to the same linear space spanned by the inter cell interference.

To this end, the following matrix is formed:

$$(\bar{\mathbf{U}}_{\mathcal{P}})_{\left[K_P(N_P-S)^+ \times \left(M_P+M_S-S\right)\right]} = \begin{pmatrix} (\mathbf{u}^1_{\mathcal{P}_1})^T\bar{\mathbf{H}}_{\mathcal{P}_1} \\ (\mathbf{u}^2_{\mathcal{P}_1})^T\bar{\mathbf{H}}_{\mathcal{P}_1} \\ \cdots \\ (\mathbf{u}^r_{\mathcal{P}_1})^T\bar{\mathbf{H}}_{\mathcal{P}_1} \\ \hline (\mathbf{u}^1_{\mathcal{P}_2})^T\bar{\mathbf{H}}_{\mathcal{P}_2} \\ (\mathbf{u}^2_{\mathcal{P}_2})^T\bar{\mathbf{H}}_{\mathcal{P}_2} \\ \cdots \\ (\mathbf{u}^r_{\mathcal{P}_2})^T\bar{\mathbf{H}}_{\mathcal{P}_2} \\ \hline \cdots \\ \hline (\mathbf{u}^1_{\mathcal{P}_{K_P}})^T\mathbf{H}_{\mathcal{P}_{K_P}} \\ (\mathbf{u}^2_{\mathcal{P}_{K_P}})^T\bar{\mathbf{H}}_{\mathcal{P}_{K_P}} \\ \cdots \\ (\mathbf{u}^r_{\mathcal{P}_{K_P}})^T\bar{\mathbf{H}}_{\mathcal{P}_{K_P}} \end{pmatrix}$$

To achieve $d_P \leq \frac{1}{L}\min\{M_P+M_S-S, K_P(N_P-S)^+\}$ sum DoF in the primary cell, without loss of generality pick the first $P = L \times d_P$ rows of $\bar{\mathbf{U}}_{\mathcal{P}}$ as:

$$\bar{\mathbf{U}}_P = \begin{pmatrix} (\bar{\mathbf{u}}_1)^T \\ \hline (\bar{\mathbf{u}}_2)^T \\ \hline \cdots \\ \hline (\bar{\mathbf{u}}_P)^T \end{pmatrix}$$

Denote the right pseudo-inverse of matrix $\bar{\mathbf{U}}_P$ by $\bar{\mathbf{V}}_P$:

$$\bar{\mathbf{V}}_P = \left[\begin{array}{c|c|c|c} \bar{\mathbf{v}}_{p_1} & \bar{\mathbf{v}}_{p_2} & \cdots & \bar{\mathbf{v}}_{p_P} \end{array}\right]$$

which means $\bar{\mathbf{U}}_P\bar{\mathbf{V}}_P = (\mathbf{I})_{P\times P}$.

For each $\bar{\mathbf{v}}_{p_i}$ denote its first M_P elements by $\mathbf{v}_{p_i}$ and the next $(M_S - S)$ ones by $\mathbf{v}'_{p_i}$. The signal transmitted from the primary BS is formed as:

$$\mathbf{x}_{\mathcal{P}} = \mathbf{x}_{p_1}\mathbf{v}_{p_1} + \mathbf{x}_{p_2}\mathbf{v}_{p_2} + \cdots + \mathbf{x}_{p_P}\mathbf{v}_{p_P},$$

where P streams of data, $\mathbf{x}_{p_1}, \mathbf{x}_{p_2} \cdots, \mathbf{x}_{p_P}$ are encoded Using a Gaussian codebook.

It remains to specify the transmission scheme from the cognitive BS. Let us define:

$$(\bar{\mathbf{H}}_{\mathcal{S}})_{[K_SN_S\times S]} = \begin{pmatrix} \bar{\mathbf{H}}_{\mathcal{S}_1} \\ \hline \bar{\mathbf{H}}_{\mathcal{S}_2} \\ \hline \cdots \\ \hline \bar{\mathbf{H}}_{\mathcal{S}_{K_S}} \end{pmatrix}$$

In order to achieve a sum DoF of d_S in the secondary cell, S streams of data are required to be reliably transmitted to the secondary users. To this end, let us denote the S rows of $\bar{\mathbf{H}}_{\mathcal{S}}$ by

$$[(\bar{\mathbf{h}}_{s_1})^T; ...; (\bar{\mathbf{h}}_{s_S})^T],$$

and form the zero forcing beam-forming vectors to the secondary users similar to a MIMO broadcast channel. That is, the beamforming vector $\mathbf{v}'_{s_i}$ is picked as the orthonormal basis of the null space of the vector space spanned by:

$$[(\bar{\mathbf{h}}_{s_1})^T; ...; (\bar{\mathbf{h}}_{s_{i-1}})^T; (\bar{\mathbf{h}}_{s_{i+1}})^T; ...; (\bar{\mathbf{h}}_{s_S})^T]$$

This is possible for $S \leq K_S \times N_S$. Next, the data stream $\mathbf{x}_{s_i}$ is encoded using dirty paper coding in $\hat{\mathbf{x}}_{s_i}$ considering $(\bar{\mathbf{h}}'_{s_i})^T\bar{\mathbf{x}}_{\mathcal{P}}$ to be the *known* interference where $(\bar{\mathbf{h}}'_{s_i})^T$ is the i-th row of $(\bar{\mathbf{H}}'_{\mathcal{S}})$ and $\bar{\mathbf{x}}_{\mathcal{P}}$ is defined as:

$$\bar{\mathbf{x}}_{\mathcal{P}} = \mathbf{x}_{p_1}\bar{\mathbf{v}}_{p_1} + \mathbf{x}_{p_2}\bar{\mathbf{v}}_{p_2} + \cdots + \mathbf{x}_{p_P}\bar{\mathbf{v}}_{p_P},$$

It should be again noted that because we assumed the secondary BS to be cognitive and have full CSI knowledge thus, $(\bar{\mathbf{h}}'_{s_i})^T\bar{\mathbf{x}}_{\mathcal{P}}$ is fully known at $\mathcal{S}$ and the secondary BS can employ dirty paper coding to cancel the interference caused by the data intended for primary mobiles on its user.
Define $\tilde{\mathbf{v}}_{p_i}$ and $\mathbf{v}_{s_j}$ as:

$$\tilde{\mathbf{v}}_{p_i} = \left[(\mathbf{v}'_{p_i})^T | \underbrace{0, \cdots, 0}_{(S)}\right]^T \quad \mathbf{v}_{s_j} = \left[\underbrace{0, \cdots, 0}_{(M_S-S)} | (\mathbf{v}'_{s_j})^T\right]^T.$$

Then, the signal transmitted by the secondary BS is equal to:

$$\mathbf{x}_{\mathcal{S}} = \sum_{i=1}^{P}\tilde{\mathbf{v}}_{p_i}\mathbf{x}_{p_i} + \sum_{j=1}^{S}\mathbf{v}_{s_j}\hat{\mathbf{x}}_{s_j}$$

Decoding: Without loss of generality, we explain the decoding of the first data streams intended for the first primary and secondary users, i.e., $\mathbf{x}_{s_1}$ and $\mathbf{x}_{p_1}$. Because we are concerned with degrees of freedom which is studied asymptotically as SNR goes to infinity, following [8] for simplicity the noise vectors are ignored. In that case, the signal received at user $\mathcal{S}_1$ is equal to:

$$\mathbf{y}_{\mathcal{S}_1} = \mathbf{H}'_{\mathcal{S}_1}\mathbf{x}_{\mathcal{P}} + \mathbf{H}_{\mathcal{S}_1}\mathbf{x}_{\mathcal{S}} = \bar{\mathbf{H}}'_{\mathcal{S}_1}\bar{\mathbf{x}}_{\mathcal{P}} + \bar{\mathbf{H}}_{\mathcal{S}_1}(\hat{\mathbf{x}}_{s_1}\mathbf{v}'_{s_1} + \cdots \hat{\mathbf{x}}_{s_S}\mathbf{v}'_{s_S})$$

Due to the choice of zero-forcing vectors, the signal received on the first dimension (first antenna on the first OFDM carrier) of user $\mathcal{S}_1$ which carries $\mathbf{x}_{s_1}$ is equal to:

$$\hat{\mathbf{x}}_{s_1} + (\bar{\mathbf{h}}'_{s_1})^T\bar{\mathbf{x}}_{\mathcal{P}},$$

and $\mathbf{x}_{s_1}$ is recovered by dirty paper decoding considering $(\bar{\mathbf{h}}'_{s_1})^T\bar{\mathbf{x}}_{\mathcal{P}}$ as the known interference.
The signal received at $\mathcal{P}_1$ is equal to:

$$\begin{aligned} &\mathbf{y}_{\mathcal{P}_1} = \mathbf{H}_{\mathcal{P}_1}\mathbf{x}_{\mathcal{P}} + \mathbf{H}'_{\mathcal{P}_1}\mathbf{x}_{\mathcal{S}} = \\ &\bar{\mathbf{H}}_{\mathcal{P}_1}\left(\mathbf{x}_{p_1}\bar{\mathbf{v}}_{p_1} + \cdots + \mathbf{x}_{p_P}\bar{\mathbf{v}}_{p_P}\right) + \\ &\bar{\mathbf{H}}'_{\mathcal{P}_1}\left(\hat{\mathbf{x}}_{s_1}\mathbf{v}'_{s_1} + \cdots + \hat{\mathbf{x}}_{s_S}\mathbf{v}'_{s_S}\right) \end{aligned}$$

Next, to decode $\mathbf{x}_{p_1}$, user $\mathcal{P}_1$ multiplies its received signal in $\mathbf{u}_1$, i.e.,

$$(\mathbf{u}_1)^T\mathbf{y}_{\mathcal{P}_1}$$

Because $\mathbf{u}_1$ is in the null space of $\bar{\mathbf{H}}'_{\mathcal{P}_1}$, the inter cell interference is canceled. By construction of $\bar{\mathbf{v}}_{p_i}$'s we have:

$$\begin{aligned} &(\mathbf{u}_1)^T\bar{\mathbf{H}}_{\mathcal{P}_1}\bar{\mathbf{v}}_{p_1} = 1, \\ &(\mathbf{u}_1)^T\bar{\mathbf{H}}_{\mathcal{P}_1}\bar{\mathbf{v}}_{p_2} = \cdots = (\mathbf{u}_1)^T\bar{\mathbf{H}}_{\mathcal{P}_1}\bar{\mathbf{v}}_{p_P} = 0 \end{aligned}$$

Thus by multiplying $(\mathbf{u}_1)^T$ in the received signal at $\mathcal{P}_1$, the inter cell interference carried by $\bar{\mathbf{H}}'_{\mathcal{P}_1}$ and the intra cell interference caused by $\bar{\mathbf{v}}_{p_2}, \cdots, \bar{\mathbf{v}}_{p_P}$ are both zero forced at the same time through the utilized interference alignment technique and $\mathbf{x}_{p_1}$ is decoded at $\mathcal{P}_1$.

Therefore, the proposed joint interference alignment and dirty paper coding (IA+DPC) achieves the sum DoF of $d_P \leq \min\{K_P(n_P - d_S)^+, (m_P + m_S - d_S)\}$ in the primary cell with a secondary sum DoF of $d_S \leq \min\{K_S n_S, m_S\}$.
Now for an irrational d_S there is a sequence of rational numbers converging to it. For each one the claimed d_P is achievable which means in the limit it will go to the same d_P which completes the proof of the theorem. ■

IV. Special cases and discussion

We begin the study of special cases by presenting an outer bound on the achievable sum DoF. Next, using the derived outer bounds we establish the optimality of our proposed scheme for a special case. The benefit of the cognitive message sharing versus the case when that is not available is also established using the following outer bound.

Theorem 2: Let $d_{\mathcal{P}_i}$ and $d_{\mathcal{S}_j}$ be an achievable DoF for mobile users $\mathcal{P}_i$ and $\mathcal{S}_j$. Then the DoF region of a cognitive cellular system satisfies the following bounds:

$$\begin{aligned}
L_1 &: d_{\mathcal{P}_i} \leq n_P, for\ 1 \leq i \leq K_P,\ d_P + d_S \leq (m_P + m_S) \\
L_2 &: d_S \leq \min\{m_S, K_S n_S\}. \\
L_3 &: d_{\mathcal{P}_i} + d_S \leq \max\{m_S, n_P\}, \quad 1 \leq i \leq K_P.
\end{aligned}$$

Proof: The bound L_1 follows from the outer bounds on the point to point MIMO channel and the fact the degrees of freedom cannot exceed the number of receive or transmit antennas. L_2 follows by assuming full cooperation between the mobile users of the secondary base station and assuming they perfectly know the interference caused by the primary base station. This cannot reduce the DoF region and L_2 follows from the outer-bounds on the DoF of the point to point MIMO channel as well.

To establish L_3, we first let $d_{\mathcal{P}_l} = 0$ for $l \neq i$ to get a bound on $d_{\mathcal{P}_i}$ and assume full cooperation at the secondary users. This reduces the problem to a MIMO interference channel (IC) with a cognitive transmitter. After the problem is reduced to a cognitive MIMO IC, we apply the sum DoF outer bound of [10] which is based on the genie aided method of [9]. It should be noted [10] assumes full and global CSI at all nodes which is not the case in our problem. However, we can assume that the extra CSI information is also provided to all nodes which does not reduce the DoF region. Using [10] we can directly show that:

$$d_{\mathcal{P}_i} + d_S \leq \max\{m_S, n_P\}$$

■

A. Special Case I

Let us consider the system with $m_P = m_S = n_P = K+1$, $n_S = 1$, $K_P = K+2$ and $K_S = K$ for an integer $K > 1$. That is a $\{K+1, K+1, K+1, 1, K+2, K\}$ cognitive system. Henceforth, this system is referred to as *example channel of type* K. Using the achievable strategy, and applying Theorem 1 the achievable region is the line connecting the following points:

$$\big(d_S = K; d_P = K+2\big)$$
$$\big(d_S = 0; d_P = 2(K+1)\big)$$

Noting that the total sum DoF of the system cannot exceed the number of transmit antennas, and the fact that $d_S = 0$ and $d_S = K$ corresponds to the corner point of the sum DoF region we can conclude that for $\{K+1, K+1, K+1, 1, K+2, K\}$ cognitive system the proposed signaling scheme is optimal in terms of DoF (i.e., in the high SNR regime).

Now, let us consider the case when cognitive message sharing is not possible. If we assume all the users in the primary and secondary cell fully cooperate with all the users in their own cells, in the absence of cognition this system reduces to a

$$\{M_1 = K+1, M_2 = K+1, N_1 = (K+2)(K+1), N_2 = K\}$$

MIMO interference channel where M_1, M_2 denote the number of antennas on the first and second transmitter and N_1, N_2 are the number of antennas on the first and second receiver respectively. This full cooperation cannot reduce the DoF region of the cellular system without cognition. Using the bound derived in [9], the maximum total sum DoF of this MIMO interference channel is equal to $K+1$ whereas our proposed scheme with cognition achieves a total sum DoF of $d_P + d_S = 2(K+1)$ which shows cognitive cooperation under our proposed scheme *strictly* outperforms the case when cognitive message sharing of the primary messages is not available to the secondary base station.

The achievable sum DoF region for the example channels of type 2 and 3 are depicted in Fig. 1.

Corollary 3: For a $\{K+1, K+1, K+1, 1, \kappa, K\}$ cognitive system with $K > 1$ achieves the optimal corner points of the sum DoF region for $2 \leq \kappa \leq K+1$.

Proof: Applying Theorem 1 at $d_S = K$, the following point is achieved by the proposed scheme:

$$(d_S = K; d_P = \kappa)$$

In other words using the proposed scheme the DoF of $d_{\mathcal{P}_i} = d_{\mathcal{S}_j} = 1$ per mobile user is achievable for this system. Note that for this system the maximum sum DoF of the cognitive cell is equal to K, which is also achieved by the proposed signaling scheme. If we assume the cognitive cell does not loose any of its DoF by helping the primary cell i.e., transmitting at $d_S = K$, and applying the bound L_3, we arrive at $d_{\mathcal{P}_i} \leq 1$. By the proposed method, $d_{\mathcal{P}_i} = 1$ is attainable. Basically, $(d_S = K; d_P = \kappa)$ is a corner point of the sum DoF region, $\mathcal{D}_{\text{sum}}$.

On the other extreme at $d_S = 0$, and applying Theorem 1 sum DoF in the primary cell is equal to $d_P = 2(K+1)$ (see also Theorem 2 for the converse) , i.e., $\mathcal{D}^{\text{in}}_{\text{sum}}$ in this case includes all the corner points of $\mathcal{D}_{\text{sum}}$. ■

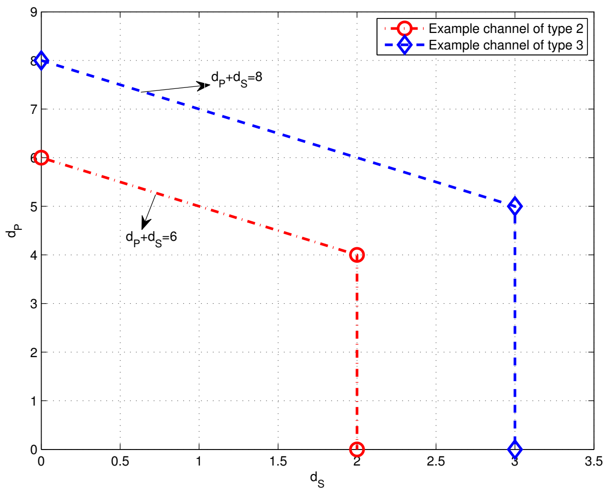

Fig. 1. Achievable sum DoF for the example channels of type 2 and 3.

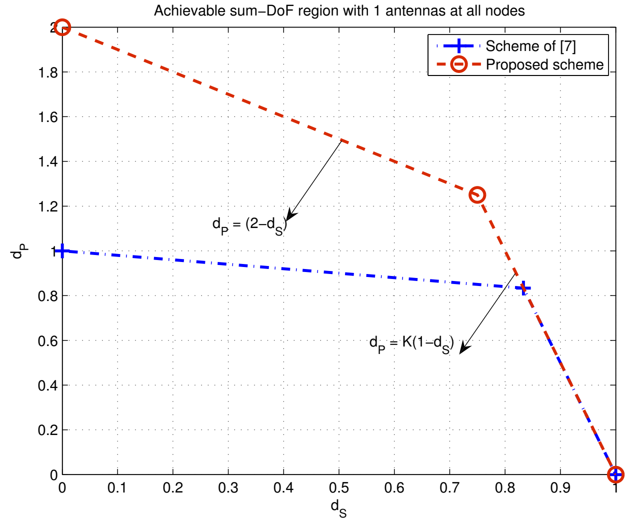

Fig. 2. Achievable sum DoF region with 1 antennas at all nodes, for $K = 5$.

B. Special Case II: One antenna at all nodes

In this subsection, we apply the result obtained in Theorem 1 to a system with $m_P = m_S = n_P = n_S = 1$ and equal number of users in each cell i.e., $K_P = K_S = K \geq 2$. This system without a cognitive base station is considered in [6] and the normalized DoF of $\frac{1}{K+1}$ per mobile user which translates to normalized sum DoF of $\frac{K}{K+1}$ per cell is achieved. That is, the point

$$(d_S = \frac{K}{K+1}, d_P = \frac{K}{K+1}), \tag{5}$$

is achieved by [6]. Our goal in this subsection is to show that point (5) is included in the sum DoF region achieved by our proposed scheme. Moreover, in [6] both of the base stations need to adapt their signaling scheme to handle their interference on the users of the other cell. However, in our proposed scheme the primary BS does not modify its transmission scheme to handle its interference on the secondary users and those are canceled by DPC.

Applying the Theorem 1 the following sum DoF region is achievable

$$\begin{aligned} 0 &\leq d_S \leq 1 \\ 0 &\leq d_P \leq \min\{(2 - d_S), K(1 - d_S)\} \end{aligned}$$

Using the above, at $d_S = \frac{K}{K+1}$ the sum DoF in primary cell is $d_P = \frac{K}{K+1}$ which means the point (5) is included in our region. Fig. 2 compares the achievable sum DoF region of our proposed scheme for the cognitive system and that of [6] for the non-cognitive counterpart.

V. Conclusions

Downlink communication for a cognitive cellular system was considered and a novel signaling scheme based on interference alignment, zero forcing and dirty paper coding was proposed. We also presented an outer bound and showed our proposed scheme to be optimal for some special cases. The benefits of the cognitive paradigm was also illustrated using the outer bound by proving the total sum DoF of the system is strictly larger than the case where cognitive message sharing is not available.

As a future work, studying other cases of CSI availability, tighter outer bounds and extending to multiple number of cells with different cognitive scenarios will be considered.

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