Resource Allocation for Throughput Enhancement in Cellular Shared Relay Networks

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***Abstract*—The downlink frame of a cellular relay network is considered, where a shared MIMO decode-and-froward relaying is used to serve the users at the edge of the cell. The relay employs zero-forcing beamforming to manage the interference among the mobile stations (MSs) at the edge of the cell. A non-cooperative scheme is considered where there is no coordination between the base stations (BSs) and the relay station (RS), and a power control algorithm for the RS is developed that maximizes the rate of the relayed users. A cooperative setting which allows the coordination of a power allocation between BSs and RSs is also considered. For this setting, based on the proposed achievable scheme, an optimization formulation is derived to maximize the total throughput of the MSs subject to a constraint on the total power of the system. The problem is solved iteratively as a sequence of geometric programs. Simulation results are provided showing that a significant increase in the network throughput can be achieved via the proposed schemes compared to a conventional cellular system with no relays.**

***Index Terms*—MIMO decode-and-forward relaying, cellular systems, power control, convex optimization.**

I. Introduction

Wireless relaying is a promising technique to enhance the capabilities of cellular wireless networks via increasing the data rate and/or robustness against channel impairments [1]. Relays are cost-effective devices that employ only a fraction of the base station (BS) functions and are not connected to the wired infrastructure of the cellular network. In addition, when a shared relay uses multiple antennas it can manage the multi-user interference via beamforming leading to an increase in the total throughput of the cellular network. Several strategies for the deployment of relays in wireless cellular networks have been considered in the literature, e.g. one-way, two-way, two-path, and shared relaying [2]. The shared relay concept was proposed in [3] where a relay is placed at the intersection of two transmitters. The shared relay is used to serve the users at the edge of these cells. Since the RS is in close proximity to these users, it can satisfy their data rate demands efficiently. Furthermore, the RS can manage the multi-user interference due to the presence of multiple MSs.

Several techniques have been proposed for managing the multi-user interference using relays. For example, a soft frequency reuse-based intercell interference coordination scheme was proposed in [4]. In [5], Kaneko et al. considered a subchannel allocation algorithm that is performed at the BS and the RS. A genetic algorithm for optimizing the system parameters was proposed in [6] in order to maximize the system spectral efficiency. These parameters include the number of RSs and their locations, the frequency-reuse pattern, and the allocated system resources. Furthermore, a distributed power allocation algorithm using the framework of game theory was proposed in [7]. In this work, the relays were modeled as rational agents engaging in a non-cooperative game where each relay node tries to maximize its individual rate while treating the signals from the other users as additive noise. In [8], Chae et al. considered MIMO relaying with a single MIMO source, a MIMO relay, and multiple single-antenna destinations, where the relay simply amplifies and forwards the received signal to the destinations. The authors of [9] have proposed a scheme that considers both the fairness of resource allocation and the system efficiency. Furthermore, the scheme takes into account that users might have different service requirements.

In this paper, we consider a cellular network with half-duplex decode-and-forward shared relaying at the intersection of each three cells as shown in Fig. 1. Each cell is divided into 3 sectors where the BS employs a single directive antenna per sector. The shared relays are equipped with multiple antennas at the intersection of the sectors of the three cells, and are used to serve the users at the cell edge. The users in each sector are assigned either to the BS serving this sector or to the shared RS based on their proximity to the BS or the RS. We focus on the downlink frame which is split into two subframes. In the first subframe, the 3 BSs send the signal desired by the users served by relay to the MIMO relay. The shared relay employs zero-forcing (or MMSE) beamforming to decode the signals transmitted by the BSs. In the second subframe, the RS also employs zero-forcing transmit beamforming to re-transmit the decoded signals to the users at the cell edge. In addition, each BS transmits to its direct-mode user in the second subframe. Our objective is to allocate the BSs and RS powers in order to maximize the total throughput of the system subject to total power constraint on the frame. We first consider a non-cooperative scheme where there is no coordination between the BSs and RS. In this case, the optimization problem reduces

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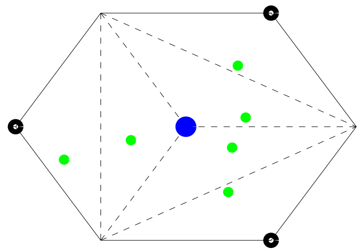

Fig. 1. System model. Black circles are BSs, green circles are users, and the blue circle is a relay node.

to a constrained waterfilling power allocation problem on the RS in order to maximize the throughput of the relayed users. We also consider a cooperative scheme where the powers of the BSs and the RS can be jointly allocated subject to a total power constraint on the frame. We derive an iterative algorithm that allocates the power to the BSs and the RS in order to maximize the total throughput of the MSs. The algorithm depends on solving a sequence of geometric programs iteratively. We show through numerical simulations that the cooperative scheme outperforms the non-cooperative one. Nevertheless, both schemes provide significant performance gains in terms of the total throughput of the network compared to a conventional cellular system that does not use relays. Unlike [5] and [8], our proposed schemes are peculiar in the sense that they require no more than one resource block for all transmissions, nor do they require spatial diversity or high complexity at the end terminals.

II. System Model and Proposed Scheme

Let us consider a single hexagonal cluster of a cellular relaying system as shown in Fig. 1. The cluster is divided into 3 sectors where each sector is served by a single antenna BS, i.e., the 3 BSs are placed at the alternate vertices of the hexagon. A single MIMO relay with M_r antennas is placed at the center of the hexagon. The MSs in each sector are served by the BS (direct users) or the RS (outer users) based on their proximity to the two stations. We consider a cellular system with frequency reuse factor of 1 and focus on *one resource block*. This resource block is utilized by 3 direct mode users (one associated with each BS) and 3 outer users that are served by the relay. Hence, each cluster has a total number of 6 single antenna MSs, where the ith sector contains two MSs; a direct MS, denoted by u_i, served by the BS of sector i, and an outer MS, denoted by e_i, served by the RS. It is worth mentioning that the scheduler can be optimized to provide multi-user diversity gain that takes into account the multiuser interference. However, this is outside the scope of this paper where we focus on interference management for cellular relay networks.

We assume that the cellular system employs time-division duplexing where the uplink and downlink transmissions occur over the same bandwidth but in different time slots. Hence, the shared RS can estimate the channel to all the MSs in the cluster with enough accuracy. For the outer users that communicate with the RS, this information can be obtained by the RS through the pilots in the signal of the outer users. Also, the RS can overhear the pilots transmitted by the direct MSs to their serving BSs, and hence, can use it to estimate the channel to these terminals.

We consider the downlink of the cellular system where each frame is divided into two subframes. In the first subframe, the BSs transmit to the RS. The received signal by the RS is given by

$$\boldsymbol{y}_r(t) = \sum_{i=1}^{3} \boldsymbol{h}_i \sqrt{p_i^{(s,1)}} w_{e_i}(t) + \boldsymbol{z}_r(t) \tag{1}$$

where $w_{e_i}(t)$ is the signal transmitted by the ith BS to the RS containing the data for the outer MS e_i, $p_i^{(s,1)}$ is the transmit power of the ith BS in the first subframe, $\boldsymbol{h}_i$ is the $M_r \times 1$ channel vector between the ith BS and the RS, and $\boldsymbol{z_r}(t)$ is the relay noise vector. The relay noise is assumed to be Gaussian zero-mean and spatially white, i.e., $E\left\{\boldsymbol{z}_r(t)\boldsymbol{z}_r^H(t)\right\} = \sigma_r^2 \boldsymbol{I}$. Here, the notation $\boldsymbol{A^H}$ denotes the conjugate transposition of the matrix $\boldsymbol{A}$.

The relay operates in half-duplex decode-and-forward mode. It employs the $3 \times M_r$ receive beamforming matrix $\boldsymbol{V}^{(1)}$ to decode the signals transmitted by the three BSs. The signal to interference-plus-noise ratio (SINR) of the output of the beamformer for the ith relayed user in the first subframe can be written as

$$\text{SINR}_{r_i} = \frac{p_i^{(s,1)} \left|\boldsymbol{v}_i^{(1)^H} \boldsymbol{h}_i\right|^2}{\sum_{j=1, j\neq i}^{3} p_j^{(s,1)} \left|\boldsymbol{v}_i^{(1)^H} \boldsymbol{h}_j\right|^2 + \sigma_r^2} \tag{2}$$

where $\boldsymbol{v}_i^{(1)^H}$ is the ith row of the matrix $\boldsymbol{V}^{(1)}$.

Although each outer user might be able to overhear its desired signal w_{e_i} in the 1^{st} phase while being transmitted to the RS, exploiting the overheard message for example via rate splitting or signal combining will add additional complexity to the MS receiver. In addition, the SINR of the link between the BSs and the outer users is expected to be significantly lower than that of the link between the RSs and the outer users. As a result, the throughput gain that can be achieved by overhearing the messages during the first phase is very small. Consequently, no form of signal combining or successive decoding at the MSs is adopted in the proposed scheme.

In the second subframe, the relay transmits to the outer MSs and the BSs transmit to the direct-mode MSs. Let $\boldsymbol{V}^{(2)} = [\boldsymbol{v}_1^{(2)}, \boldsymbol{v}_2^{(2)}, \boldsymbol{v}_3^{(2)}]$ denote the transmit beamfoming matrix of the RS in the second subframe. Note that the transmit beamforming vectors are normalized such that $\|\boldsymbol{v}_i^{(2)}\| = 1$. Therefore, the signal transmitted from the RS in the second subframe can be expressed as

$$\boldsymbol{x}_r(t) = \sum_{i=1}^{3} \boldsymbol{v}_i^{(2)} \sqrt{p_i^{(r)}} w_{e_i}(t) \tag{3}$$

where $p_i^{(r)}$ is the power used by the RS to transmit the message of the ith outer MS, e_i. Concurrently, each BS transmits the signal intended to its direct user. The signal transmitted by the ith BS to its direct user is given by

$$x_{u_i}(t) = \sqrt{p_i^{(s,2)}}\, w_{u_i}(t) \quad (4)$$

where $w_{u_i}(t)$ is the signal carrying the data for the direct MS of the ith BS and $p_i^{(s,2)}$ is the power transmitted by the ith BS in the second subframe. As a result, the signal received at the ith outer user is given by

$$y_{e_i}(t) = \boldsymbol{g}_{(e_i,r)}^H \boldsymbol{x}_r(t) + \boldsymbol{g}_{(e_i,s)}^H \boldsymbol{x}_u(t) + z_{e_i}(t) \quad (5)$$

where $\boldsymbol{x}_u(t)$ is the vector containing all the signals transmitted from the 3 BSs in the second subframe, i.e., $\boldsymbol{x}_u(t) = [x_{u_1}(t), x_{u_2}(t), x_{u_3}(t)]^T$, $\boldsymbol{g}_{(e_i,r)}$ is the $M_r \times 1$ channel vector between the relay and the ith outer user, $\boldsymbol{g}_{(e_i,s)}$ is the channel vector between the BSs and the ith outer user, and $z_{e_i}(t)$ is the zero-mean Gaussian noise generated at the ith relayed user whose variance is given by $\sigma_{e_i}^2 = \mathrm{E}\{|z_{e_i}(t)|^2\}$. As a result, the received SINR of the ith outer MS can be expressed as

$$\mathrm{SINR}_{e_i} = \frac{p_i^{(r)} \left|\boldsymbol{g}_{(e_i,r)}^H \boldsymbol{v}_i^{(2)}\right|^2}{\sum_{j=1, j\neq i}^{3} p_j^{(r)} \left|\boldsymbol{g}_{(e_i,r)}^H \boldsymbol{v}_j^{(2)}\right|^2 + \sum_{k=1}^{3} p_k^{(s,2)} \left|g_{(e_i,s_k)}\right|^2 + \sigma_{e_i}^2} \quad (6)$$

where $g_{(e_i,s_k)}$ is the kth element of $\boldsymbol{g}_{(e_i,s)}$. On the other hand, the signal received at the ith direct user is given by

$$y_{u_i}(t) = \boldsymbol{g}_{(u_i,s)}^H \boldsymbol{x}_u(t) + \boldsymbol{g}_{(u_i,r)}^H \boldsymbol{x}_r(t) + z_{u_i}(t) \quad (7)$$

where $\boldsymbol{g}_{(u_i,s)}$ is the channel vector between the BSs and the ith direct user, $\boldsymbol{g}_{(u_i,r)}$ is the $M_r \times 1$ channel vector between the relay and the ith direct user and $z_{u_i}(t)$ is the noise generated at the ith direct user which is also assumed to be zero-mean Gaussian with variance $\sigma_{u_i}^2$. As a result, the received SINR at the ith direct user can be expressed as

$$\mathrm{SINR}_{u_i} = \frac{p_i^{(s,2)} \left|g_{(u_i,s_i)}\right|^2}{\sum_{j=1}^{3} p_j^{(r)} \left|\boldsymbol{g}_{(u_i,r)}^H \boldsymbol{v}_j^{(2)}\right|^2 + \sum_{k=1, k\neq i}^{3} p_k^{(s,2)} \left|g_{(u_i,s_k)}\right|^2 + \sigma_{u_i}^2} \quad (8)$$

where $g_{(u_i,s_k)}$ is the kth element of the $\boldsymbol{g}_{(u_i,s)}^H$ row vector.

III. Optimization Formulation

In this section, we consider the problem of designing the relay beamforming matrices, i.e., $\boldsymbol{V}^{(1)}$ and $\boldsymbol{V}^{(2)}$, and allocating the power to the BSs and the RS in the two subframes. The design objective is to maximize the sum rate of the MSs. Since we compare the performance of the proposed algorithms with a conventional cellular system with no relays, for the sake of fairness, we assume that the total power budget consumed per cell in each frame is given by P_{max}[1]. Hence, we can write the relay beamforming and power allocation problem as

$$\begin{aligned} \max_{\{p_i^{(s,1)}, p_i^{(s,2)}, p_i^{(r)}\}} \quad & \sum_{i=1}^{3} \min\{R_{e_i,1}, R_{e_i,2}\} + R_{u_i} \\ \text{s.t.} \quad & \sum_{i=1}^{3} p_i^{(s,1)} + p_i^{(s,2)} + p_i^{(r)} \leq P_{\max} \end{aligned} \quad (9)$$

where $R_{e_i,1}$ and $R_{e_i,2}$ are the rates of the ith outer user in the first and the second subframes, respectively, whereas R_{u_i} is the rate of the ith direct-mode user and $P_{\max}$ denotes the total power budget of the system per frame.

We propose two schemes to solve this optimization problem. The first scheme assumes that there is no coordination between the transmitting terminals (BSs and RS) while the other scheme considers the case when the shared relay can coordinate its transmission power with the BSs.

A. Non-cooperative scheme

In this scheme, we assume that there is no coordination among the BSs and/or the RS. As a result, the power is divided equally between the transmitting terminals in the two subframes. Hence, in the first subframe the transmission power of the ith BS is given by $p_i^{(s,1)} = P_{\max}/9$. In the next subframe, the transmission power of the ith BS is also given by $p_i^{(s,2)} = P_{\max}/9$, whereas, the maximum transmission power of the RS is constrained such that

$$\sum_{i=1}^{3} p_i^{(r)} \leq \frac{P_{\max}}{3}. \quad (10)$$

First, we consider the design of the beamforming vectors of the RS. In order to enable the shared relay to handle the multi-user interference of the three cells, we assume that the number of RS antennas is larger than or equal to the number of spatially-multiplexed MSs in the three cells, i.e., $M_r \geq 6$. Assuming that the BSs employ Gaussian codebooks, the rate of transmission from the ith BS to the RS in the first subframe is given by $R_{e_i,1} = \log(1 + \mathrm{SINR}_{r_i})$. Hence, this rate can be maximized by selecting the receive beamforming vectors of the RS such that the output SINR is maximized [10] by setting

$$\boldsymbol{v}_i^{(1)} = \left(\sum_{j=1, j\neq i}^{3} p_j^{(s,1)} \boldsymbol{h}_j \boldsymbol{h}_j^H + \sigma_r^2 \boldsymbol{I}\right)^{-1} \boldsymbol{h}_i. \quad (11)$$

In the second subframe, the relay beamforming vectors are selected according to the zero-forcing (ZF) criterion. Let

$$\tilde{\boldsymbol{v}}_i^{(2)} = \left(\boldsymbol{I} - \tilde{\boldsymbol{G}}_i \left(\tilde{\boldsymbol{G}}_i^H \tilde{\boldsymbol{G}}_i\right)^{-1} \tilde{\boldsymbol{G}}_i^H\right) \boldsymbol{g}_{(e_i,r)} \quad (12)$$

where the columns of the $M_r \times 5$ matrix $\tilde{\boldsymbol{G}}_i$ are given by $\{\{\boldsymbol{g}_{(e_i,r)}\}_{k\neq i}, \{\boldsymbol{g}_{(u_i,r)}\}_{k=1}^3\}$, the relay beamforming vector for the ith MS is then given by

$$\boldsymbol{v}_i^{(2)} = \tilde{\boldsymbol{v}}_i^{(2)} / \|\tilde{\boldsymbol{v}}_i^{(2)}\| \quad (13)$$

and hence, the relay does not cause any interference to the outer MS or direct MSs inside the cluster.

Our objective now is to determine the power allocated to the streams transmitted by the RS during the second subframe in order to maximize the sum rate of the outer MSs subject to a power constraint on the RS. Hence, the optimization problem in (9) reduces to

$$\begin{aligned} \max_{\{p_i^{(r)}\}_{i=1}^3} \quad & \sum_{i=1}^{3} R_{e_i,2} \\ \text{s.t.} \quad & \sum_{i=1}^{3} p_i^{(r)} \leq \frac{P_{\max}}{3} \\ & R_{e_i,2} \leq R_{e_i,1} \quad \forall i = 1, \ldots, 3 \end{aligned} \quad (14)$$

where $R_{e_i,2} = \log(1 + \mathrm{SINR}_{e_i})$. Note that the last constraint in (14) is due to the fact that the maximum rate that can

[1]In the non-cooperative scheme, we assume that each transmitting terminal has an instantaneous power constraint and the total power P_{max} is divided equally between the transmitting terminals.

be achieved by the outer MSs is constrained by the rate transmitted from the BS to the RS in the first subframe. Substituting with (13) in (6), we can write (14) as

$$\begin{aligned}
\max_{\left\{p_i^{(r)}\right\}_{i=1}^{3}} \quad & \sum_{i=1}^{3} \log\left(1+\frac{p_i^{(r)}\left|\boldsymbol{g}_{(e_i,r)}^{H}\boldsymbol{v}_i^{(2)}\right|^2}{\sum_{k=1}^{3} p_k^{(s,2)}\left|g_{(e_i,s_k)}\right|^2+\sigma_{e_i}^2}\right) \\
\text{s.t.} \quad & \log\left(1+\frac{p_i^{(r)}\left|\boldsymbol{g}_{(e_i,r)}^{H}\boldsymbol{v}_i^{(2)}\right|^2}{\sum_{k=1}^{3} p_k^{(s,2)}\left|g_{(e_i,s_k)}\right|^2+\sigma_{e_i}^2}\right) \leq R_{e_i,1} \quad \forall i \\
& \sum_{i=1}^{3} p_i^{(r)} \leq \frac{P_{\max}}{3}.
\end{aligned} \tag{15}$$

Note that the rates $R_{e_i,1}$ are fixed as they are determined by the powers allocated to the BSs during the first subframe. The above problem is a constrained waterfilling problem [10] that can be solved efficiently using interior-point methods [11].

B. Cooperative scheme

Due to the absence of any form of coordination between the RS and the BSs, the transmission powers of different stations are determined independently. This leads to a loss of performance, e.g., since the rates of the outer MSs are completely determined by the transmission power of the BSs in the first subframe. In this subsection, we assume that the RS and the BSs can coordinate their transmission powers jointly in order to maximize the total throughput of the network. The RS takes over the task of coordinating with the BSs in order to efficiently utilize the power budget. It is worth mentioning that the only requirement of the cooperative scheme over the non-cooperative one is the capability of the RS to obtain the gains of the channels between the direct users and the BSs. This information is available in the feedback information (such as the channel quality indicator (CQI)) sent by each direct user to its BS in control messages. Thereby, if the RS successfully overhears these control messages, the cooperative scheme can be applied without the need of any additional feedback messages between the RS and the BSs.

Since the main function of the shared relay is to manage the multi-user interference, the RS employs ZF receive beamforming in the first subframe by selecting the beamforming vectors as

$$\boldsymbol{v}_i^{(1)} = \left(\boldsymbol{I} - \tilde{\boldsymbol{H}}_i\left(\tilde{\boldsymbol{H}}_i^{H}\tilde{\boldsymbol{H}}_i\right)^{-1}\tilde{\boldsymbol{H}}_i^{H}\right)\boldsymbol{h}_i, \tag{16}$$

where $\tilde{\boldsymbol{H}}_i$ is the $M_r \times 2$ matrix whose columns are $\{\boldsymbol{h}_k\}_{k\neq i}$. Also, in the second subframe, the RS employs the ZF transmit beamforming vectors of (13). Using (13) and (16), we can write the optimization problem in (9) as

$$\begin{aligned}
\max \quad & \Pi_{i=1}^{3}\left(1+\tau_i\right)\left(1+\mathrm{SINR}_{u_i}\right) \\
\text{s.t.} \quad & \frac{p_i^{(s,1)}}{\sigma_r^2}\left|\boldsymbol{v}_i^{(1)H}\boldsymbol{h}_i\right|^2 \geq \tau_i \quad \forall i, \\
& \frac{p_i^{(r)}\left|\boldsymbol{g}_{(e_i,r)}^{H}\boldsymbol{v}_i^{(2)}\right|^2}{\sum_{k=1}^{3} p_k^{(s,2)}\left|g_{(e_i,s_k)}\right|^2+\sigma_{e_i}^2} \geq \tau_i \quad \forall i, \\
& \frac{p_i^{(s,2)}\left|g_{(u_i,s_i)}\right|^2}{\sum_{k=1,k\neq i}^{3} p_k^{(s,2)}\left|g_{(u_i,s_k)}\right|^2+\sigma_{u_i}^2} \geq \mathrm{SINR}_{u_i} \quad \forall i, \\
& \sum_{i=1}^{3} p_i^{(s,1)}+p_i^{(s,2)}+p_i^{(r)} \leq P_{\max}
\end{aligned} \tag{17}$$

where we have used the auxiliary optimization variables $\{\tau_i, \mathrm{SINR}_{u_i}\}_{i=1}^{3}$ in addition to the original variables $\{p_i^{(s,1)}, p_i^{(s,2)}, p_i^{(r)}\}_{i=1}^{3}$. Note that on taking the exponentiation for the original optimization problem in (9), the log terms are removed and will result in the multiplication form shown above. Now, the constraints in the problem can be written in the form of posynomial inequality constraints, i.e., $f_k(\boldsymbol{z}) \leq 1$ where $\boldsymbol{z}$ is a vector containing the optimization variables. However, the objective function is not a monomial [11], and hence, the optimization problem in (17) is not an extended Geometric Program. Hence, in order to be able to solve (17) efficiently, we approximate the objective function as [12]

$$\prod_{i=1}^{3}\left(1+\tau_i\right)\left(1+\mathrm{SINR}_{u_i}\right) \approx c\prod_{i=1}^{3}\left(\tau_i\right)^{\lambda_i^e}\left(\mathrm{SINR}_{u_i}\right)^{\lambda_i^u} \tag{18}$$

where $c > 0$ and $\{\lambda_i^u, \lambda_i^e\}_{i=1}^{3}$ are constants and are given by

$$\begin{aligned}
\lambda_i^e &= \frac{\tau_i}{1+\tau_i} \quad \forall i, \\
\lambda_i^u &= \frac{\mathrm{SINR}_{u_i}}{1+\mathrm{SINR}_{u_i}} \quad \forall i, \\
c &= \frac{\prod_{i=1}^{3}\left(\tau_i+1\right)\left(\mathrm{SINR}_{u_i}+1\right)}{\prod_{i=1}^{3}\left(\tau_i\right)^{\lambda_i^e}\left(\mathrm{SINR}_{u_i}\right)^{\lambda_i^u}}.
\end{aligned} \tag{19}$$

Using the above approximation in (18), the optimization problem in (17) is solved with a sequence of GPs via the following iterative procedure:

- Initialize c and $\{\lambda_i^u, \lambda_i^e\}_{i=1}^{3}$ randomly.
- **while** target accuracy is not reached

  do solve (17) using the approximation in (18).
  Update c and $\{\lambda_i^u, \lambda_i^e\}_{i=1}^{3}$ from (19).

  end

Note that as proven in [12], the objective function in (17) increases after every iteration and the algorithm is guaranteed to converge to a local optimum point. However, convergence to a global optimum point is not guaranteed as the problem is not convex.

IV. Numerical Simulations

In this section, we present some numerical results to demonstrate the performance of the proposed schemes. Simulation results are averaged over 100 runs. In each run, each channel coefficient is given by $A_G S_G P L_G H_F$ where H_F denotes the

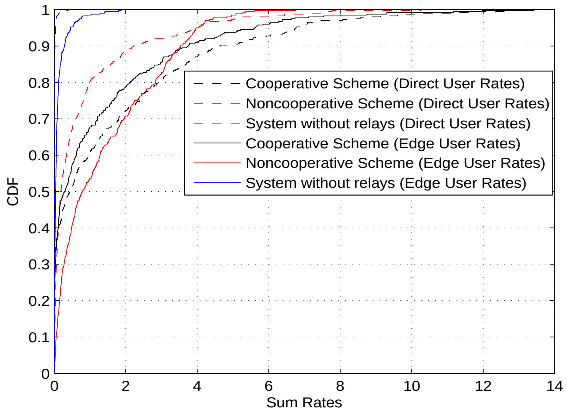

Fig. 2. Empirical CDF of the throughput of one cluster at $P_{\max} = 40$ dB.

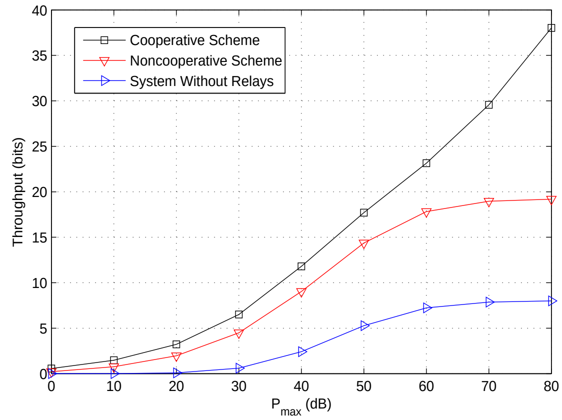

Fig. 3. Total throughput of one cluster versus frame power budget $P_{\max}$.

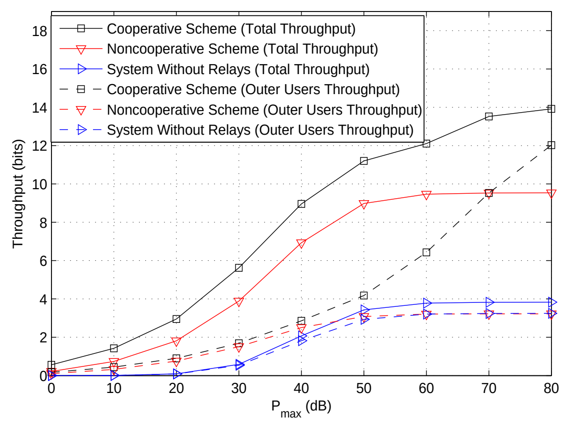

Fig. 4. Total throughput of one cluster versus frame power budget $P_{\max}$ for a system composed of 19 clusters.

fast fading coefficient of the channel, while A_G, S_G and PL_G represent the antenna pattern gain, the shadowing gain and the pathloss gain, respectively. The relay has omni-directional antennas whose gain is unity, i.e, $A_G = 1$, while the BS employs a directional antenna in each sector with gain (in decibels) given by $A_L(\Delta\theta) = -\min\left\{12\left(\frac{|\Delta\theta|}{\theta_{3\text{dB}}}\right)^2, 20\right\}$ where $\theta_{3\text{dB}} = 70^\circ$ corresponds to the 3dB beamwidth of the BS antenna, and $\Delta\theta$ is the angular direction of the MS with respect to the mid-sector direction. The shadowing gain, S_G, is modelled as log-normal with standard deviation 8 dB for the channel between a BS and MS and the channel between the RS and the MS, whereas the standard deviation of the shadowing loss for the channel between the BS and the RS is 6 dB. The path loss, PL_G, is calculated according to the IEEE 802.16j model [13], where we assume the BS and relay antennas are 30 m and 15 m high, respectively, and the MS antennas are located at a height of 1 m. The fast fading coefficient, H_F, is modelled as zero-mean complex Gaussian random variable with unit variance.

In this simulation, we assume the cell radius is 1000 m. The relay has $M_r = 6$ antennas. The noise variances, σ_r^2, $\sigma_{e_i}^2$ and $\sigma_{u_i}^2$, are assumed to be -144 dB. In each run, we generate 6 users randomly in each hexagonal cell such that 1 MS exists in each subsector of Fig. 1. We compare the proposed schemes with a conventional cellular system that uses time-division multiplexing and does not employ any relays. For this system, each BS serves an MS in one subframe and the power is divided equally between the 3 BSs. Hence, $p_i^{(s,1)} = p_i^{(s,2)} = \frac{P_{\max}}{6}$. The first subframe is assigned to serve the 3 users in the subsectors close to the BSs while in the second subframe the three outer users are served.

First, we consider a system composed of only one cluster. Fig. 2 shows the cumulative distribution function (CDF) of the total direct and outer users' throughput achieved using the proposed schemes for a total power budget per frame given by $P_{\max} = 40$ dB, where 400 channel realizations are used. We note that, for outer users, there is a crossing in the CDF achieved by the two proposed schemes. This can be explained that for outer users, the cooperative scheme has higher variation in throughput with respect to the channel conditions. This is because the cooperative scheme jointly optimizes the transmitted power for the outer users in the two phases. Hence, it is relatively more sensitive to channel variations than the non-cooperative scheme, which optimizes the 2nd phase power only, resulting in a higher variance of the achieved rates. Fig. 3 shows the sum rates of the system obtained by the cooperative and the non-cooperative schemes versus different values of frame power constraint $P_{\max}$. We can see from the two figures that the cooperative scheme outperforms the non-cooperative one and that both schemes yield significant performance improvements over a conventional system without relays.

Next, we consider a system consisting of 19 clusters in a wrap around arrangement where all the clusters use the same frequency, i.e., a unity frequency reuse factor is employed. Note that the proposed schemes allocate the power to the terminals of each cluster independently and do not take into account the out-of-cluster interference. Fig. 4 shows the total throughput of the central cluster versus $P_{\max}$. It also shows the throughput of the outer MSs. We can see from this figure

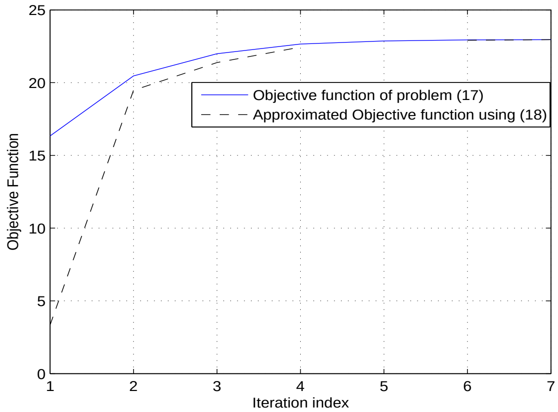

Fig. 5. The original and approximated objective functions versus iteration index of the proposed iterative algorithm at $P_{\max} = 50$ dB.

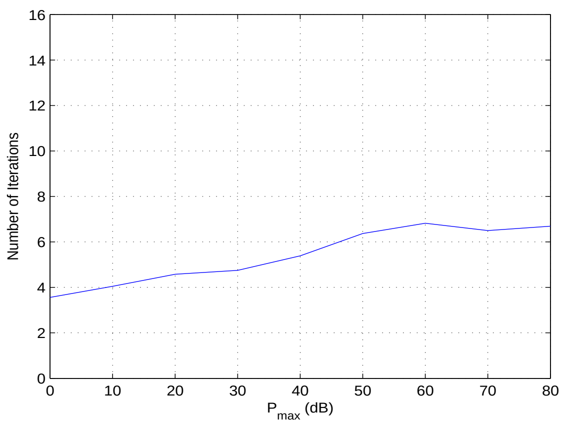

Fig. 6. Average number of iterations of the proposed iterative algorithm versus frame power budget $P_{\max}$.

that the out-of-cluster interference reduces the throughput of the network. Nevertheless, the performance of the proposed schemes is significantly superior to that of a conventional system which does not employ any relays. We can also see from this figure that the cooperative scheme enhances the rates of the outer users significantly.

Finally, we investigate the convergence of the iterative algorithm. Fig. 5 shows the original objective function in (17) and the approximated one in (18) at each iteration of the iterative algorithm. The value of $P_{\max}$ is chosen as 50. We can notice that both functions increase with each iteration till the approximate function, which is considered a lower bound, becomes within 0.01 tolerance from the original one, and hence, the algorithm terminates providing the solution of the approximated problem which is very close to the solution of the problem in (17). Fig. 6 shows the average number of iterations required for convergence of the iterative algorithm versus the different values of frame power constraint $P_{\max}$. The algorithm terminates and convergence is declared when the approximation in (18) becomes within 0.01 tolerance. Simulation results are averaged over 100 channel realizations. We can see from this figure that the number of required iterations does not increase significantly with increasing $P_{\max}$ and that at most 7 iterations are sufficient for convergence.

V. Conclusion

We have investigated the use of shared relaying in cellular networks in order to maximize the system throughput by managing the interference among the users of the cell. We have considered a non-cooperative scheme where there is no coordination between the BSs and/or the relay. For this scheme, we have proposed a power allocation algorithm for the relay that can be considered a constrained waterfilling algorithm. We have also proposed a joint power allocation algorithm for the BSs and the RS, where this coordination does not imply the use of the network backhaul. The proposed algorithm obtains the power allocation by solving a sequence of Geometric Programs that is guaranteed to converge. In addition, the complexity of the proposed schemes is not prohibitively high. We have shown through numerical simulations that the performance of the cooperative scheme is superior to that of the non-cooperative one. We have also shown that adding relays to the system enhances the system performance in terms of total throughput by increasing the cell-edge user rates.

References

[1] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bölcskei, "An overview of MIMO communications – A key to Gigabit wireless," *Proc. IEEE*, vol. 92, pp. 198–218, Feb. 2004.

[2] S. W. Peters, A. Y. Panah, K. T. Truong, and R. W. Heath, "Relay architectures for 3GPP LTE-advanced," *EURASIP Journal on Wireless Communication and Networking*, vol. 2009, pp. 1–14, March 2009.

[3] A. Tajer and A. Nosratinia, "A broadcasting relay for orthogonal multiuser channels," *IEEE Globecom*, Nov. 2006.

[4] J. Liu, D. Wang, J. Pang, J. Wang, and G. Shen, "Inter-cell interference coordination based on soft frequency reuse for relay enhanced cellular network," *Proc. IEEE Personal, Indoor and Mobile Radio Communications*, 2010.

[5] M. Kaneko and P. Popovski, "Radio resource allocation algorithm for relay-aided cellular OFDMA system," in *Proceedings of the IEEE International Conference on Communications (ICC '07)*, Glasgow, Scotland, UK, June 2007.

[6] W-H Sheen, S-J Lin, and C-C Huang, "Downlink optimization and performance of relay-assisted cellular networks in multicell environments," *IEEE Trans. Veh. Technol.*, vol. 59, no. 5, June 2010.

[7] S. Ren and M. van der Schaar, "Distributed power allocation in multi-user multi-channel cellular relay networks," *IEEE Transactions on Wireless Communication*, vol. 9, no. 6, June 2010.

[8] C. Chae, T. Tang, R. W. Heath, and S. Cho, "Mimo relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Transactions on Signal Processing*, vol. 56, no. 2, pp. 727–738, 2008.

[9] K. Chen, B. Zhang, D. Liu, J. Li, and G. Yue, "Fair resource allocation in OFDMA two-hop cooperative relaying cellular networks," *Proc. IEEE Vehicular Technology Conference*, Sept 2009.

[10] D. Tse and P. Viswanath, *Fundamental of Wireless Communication*, Cambridge University Press, Cambridge, UK, 2005.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, Cambridge, UK, 2004.

[12] J. Tadrous, A. Sultan, M. Nafie, and A. El-Keyi, "Power control for constrained throughput maximization in spectrum shared networks," *IEEE Globecom*, Dec. 2010.

[13] *Multi-hop relay system evaluation methodology (Channel model and performance metric)*, IEEE 802.16j-06/013r3 Std., Feb. 2007.